Name of College – S. S. College J. Bad

Deptt – Mathematics

Topic – Contd Problem Based on System of Circles, Radical Axis. and Co-axial Circles

Class – B. Sc (I) – sub

Date – 07-07-2020

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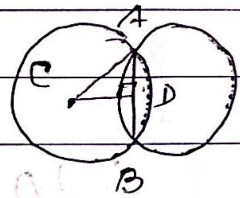
Problem → Find the radical axis and the length of common chord of two circles

$$x^2+y^2+3x+5y+4=0$$
and $$x^2+y^2+5x+3y+4=0.$$



Sol → The Given Circles are:

$$x^2+y^2+3x+5y+4=0 \quad \text{——— (I)}$$
$$x^2+y^2+5x+3y+4=0 \quad \text{——— (II)}$$

The Radical axis of (I) and (II) is Given by

$$S_1 - S_2 = 0$$
$$\Rightarrow (x^2+y^2+3x+5y+4)-(x^2+y^2+5x+3y+4)=0$$
$$\Rightarrow -2x+2y=0$$
$$\Rightarrow x-y=0 \quad \text{——— (III)}$$

Centre C of (I) is
$$(-3/2, -5/2)$$
and radius is
$$\sqrt{9/4+\frac{25}{4}-4} = \sqrt{\frac{9+25-16}{4}}$$
$$= \sqrt{\frac{18}{4}} = \sqrt{9/2} = \frac{3}{\sqrt{2}}$$

$$CD = \frac{-3/2 - (-5/2)}{\sqrt{1^2 + (-1)^2}}$$ Where CD is perpendicular Distance of AB from C.

$$= \frac{1}{\sqrt{2}}$$

AC = radius of (1) $= \frac{3}{\sqrt{2}}$

$$\therefore AC^2 = CD^2 + AD^2$$

$$AD^2 = CD^2 - AC^2$$

$$= \frac{9}{2} - \frac{1}{2} = 4$$

$$\therefore AD = 2$$

Hence the Required length of Common chord.

$$= AB = 2 \times AD = 2 \times 2 = 4.$$

Problem 2.

Find the Equation of the Circle whose diameter is the Common chord of the Circles

$$x^2 + y^2 + 2x + 3y + 1 = 0$$

and $x^2 + y^2 + 4x + 3y + 2 = 0$

Solution → the Equation of Common chord of the circles,

$$x^2 + y^2 + 2x + 3y + 1 = 0 \quad \text{— (I)}$$
$$x^2 + y^2 + 4x + 3y + 2 = 0 \quad \text{— (II)}$$

is given by $(x^2+y^2+2x+3y+1) - (x^2+y^2+4x+3y+2) = 0$

(using $S_1 - S_2 = 0$ is the Equation of common chord)

$-2x - 1 = 0 \Rightarrow$

$2x + 1 = 0$ —— (II)

Now the equation of circle passing through (I) and (II) is

$x^2 + y^2 + 2x + 3y + 1 + K(2x+1) = 0$

using Equation

[of circle passing through a given Circle S_1 and given st line L.]

$\Rightarrow x^2 + y^2 + (K+1)2x + 3y + K + 1 = 0$ —— (III)

Whose Centre is $\left[-(K+1), -3/2\right]$

If the Common Chord is the a diameter then the Centre

$(-K-1, -3/2)$ lies on (II)

there for $2(-K-1) + 1 = 0$

$\Rightarrow -2K - 1 = 0$

$K = -1/2$

putting these value of $K = -1/2$ in (III) the equation of required circle is

$x^2 + y^2 + 2(-1/2 + 1)x + 3y - 1/2 + 1 = 0$

$x^2 + y^2 + \cancel{2} \times \frac{1}{\cancel{2}} x + 3y + 1/2 = 0$

$2x^2 + 2y^2 + 2x + 6y + 1 = 0$

Problem 3.

Find the equation to the circle whose diameter is common chord of

$$(x-a)^2+y^2=a^2$$

and $$x^2+(y-b)^2=b^2.$$

Also find the length of the common chord.

Solution :→ The given circles are

$$(x-a)^2+y^2=a^2 \quad \text{——} \ (\text{I})$$

and $$x^2+(y-b)^2=b^2 \quad \text{——} \ (\text{II})$$

(I) − (II)

$$(x-a)^2+y^2-x^2-(y-b)^2=a^2-b^2$$

$$\cancel{x^2-2ax+a^2+y^2-x^2-y^2+2by+b^2=a^2-b^2}$$

$$\cancel{\Rightarrow -2ax+2by=-2b^2}$$

$$-ax=-b^2-by$$

$$\cancel{x^2}-2ax+\cancel{a^2}+\cancel{y^2}-\cancel{x^2}-\cancel{y^2}+2by\cancel{+b^2}=\cancel{a^2}\cancel{-b^2}$$

$$-2ax+2by=0$$

$$\Rightarrow y=\frac{ax}{b} \quad \text{——} \ (\text{III})$$

From (I)

$$(x-a)^2+\frac{a^2x^2}{b^2}=a^2$$

$$\Rightarrow x^2-2ax+a^2+\frac{a^2x^2}{b^2}=a^2$$

$$\Rightarrow x\left[x-2a+\frac{a^2x}{b^2}\right]=0$$

$\therefore x = 0$ or $x\left(1 + \frac{a^2}{b^2}\right) - 2a = 0$

$$\Rightarrow x\left(\frac{a^2+b^2}{b^2}\right) = 2a$$

$$\Rightarrow x = \frac{2ab^2}{a^2+b^2}$$

$\therefore$ From $y = \frac{ax}{b}$

When $x = 0$, $y = 0$ $\quad x = \frac{2ab^2}{a^2+b^2}$

$$y = \frac{a}{b} \cdot \frac{2ab^2}{a^2+b^2} = \frac{2a^2b}{a^2+b^2}$$

Therefore point of intersection of (I) and (II) are

$(0,0)$ and $\left(\frac{2ab^2}{a^2+b^2}, \frac{2a^2b}{a^2+b^2}\right)$

Thus the equation of the circle. The end point of whose diameter is $(0,0)$ and $\left(\frac{2ab^2}{a^2+b^2}, \frac{2a^2b}{a^2+b^2}\right)$ is Given

by using

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2) = 0$$

$$\Rightarrow x\left(x - \frac{2ab^2}{a^2+b^2}\right) + y\left(y - \frac{2a^2b}{a^2+b^2}\right) = 0$$

$$\Rightarrow (x^2+y^2)(a^2+b^2) = 2ab(bx+ay)$$

And length of common chord

$$= \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$$

$$= \sqrt{\left(0 - \frac{2ab^2}{a^2+b^2}\right)^2 + \left(0 - \frac{2a^2b}{a^2+b^2}\right)^2}$$

$$= \frac{2ab}{\sqrt{a^2+b^2}}$$